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Heat transfer between two opposed non-isothermal counter-rotating jets

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1. INTRODUCTION

Momentum and heat transfers in rotating flow fields are of practical interest due to the relevance to some industrial devices such as rotating machinery and heat exchangers. Compared with the straining effect, less attention was paid to the effect of rotation on fluid mechanics and heat transfer. Recently the effect of counter-rotation on opposed jets has received considerable attention in the field of combustion [1–5] because the opposed counter-rotating jets are typical flows to analyze the effects of stretch on flames. However, for mathematical simplicity, some crucial assumptions such as approximations of constant density and thermodynamic properties, negligible viscous effect, and the same temperature at the exits of two opposed jets, were made in these studies.

The process of heat transfer was absent due to the lack of temperature gradient between two jets [5]. Consequently, the aspects of fluid mechanics and heat transfer obtained from the model of isothermal (incompressible) counter-rotating jets are obviously inadequate from a fundamental point of view. Further investigation of the flow field and heat transfer for non-isothermal counter-rotating jets is necessary because this issue is significant in the non-isothermal turbulent modeling.

2. ANALYSIS

For low-Mach-number, non-isothermal and axisymmetric viscous opposed jets of infinite radial extent and finite axial separation distance under counter-rotation (the inset of Fig. 1), the appropriate governing equations and boundary conditions are well known.

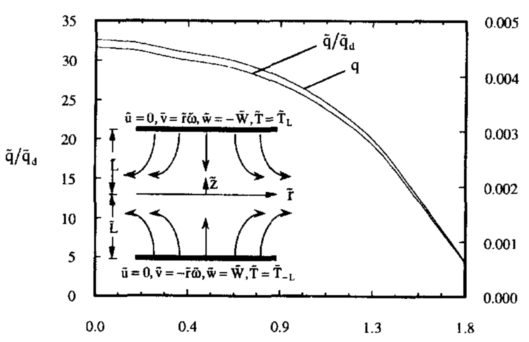

Fig. 1. $\tilde{q}/\tilde{q}_d$ and q vs ω ($Re_L = 600$, $Pr = 1$ and $\tilde{T}_{-L} = 0.8$).

The dimensionless variables and parameters are introduced as follows

$$r = \tilde{r}/\tilde{L}, \quad z = \tilde{z}/\tilde{L}, \quad \rho = \tilde{\rho}, \quad \tilde{\rho}_L, \quad \vec{v} = \tilde{\vec{v}}/\tilde{W},$$
$$P = \tilde{P}/\tilde{\rho}_L\tilde{W}^2, \quad T = \tilde{T}/\tilde{T}_L, \quad \mu = \tilde{\mu}/\tilde{\mu}_L,$$
$$\lambda = \tilde{\lambda}/\tilde{\lambda}_L, \quad Re_L = \tilde{\rho}_L\tilde{W}\tilde{L}/\tilde{\mu}_L,$$
$$Pr = \tilde{c}_p\tilde{\mu}_L/\tilde{\lambda}_L, \quad \omega = \tilde{\omega}\tilde{L}/\tilde{W} \tag{1}$$

The transformed coordinate ξ is defined as

$$\xi = \int_{-1}^{z} \rho\,dz \Big/ \int_{-1}^{1} \rho\,dz \tag{2}$$

In terms of the coordinate ξ, the problem of interest admits similar solutions

$$\rho = \rho(\xi), \quad u = rF(\xi)/2, \quad v = rH(\xi),$$
$$w = G(\xi)/\rho, \quad P = br^2/2 + h(\xi), \quad T = T(\xi) \tag{3}$$

that satisfy the following dimensionless equations

$$G' + cF = 0 \tag{4}$$

$$Re_L^{-1}F'' + 2c^2H^2 = 2bc^2T + cF'G + c^2F^2/2 \tag{5}$$

$$Re_L^{-1}H'' = c^2FH + cGH' \tag{6}$$

$$Pr^{-1}Re_L^{-1}T'' = cGT' \tag{7}$$

$$\rho T = 1 \tag{8}$$

with the boundary conditions

$$F = 0, \quad H = Ro^{-1} = \omega,$$
$$G = -1, \quad T = 1 \quad \text{at}\ \xi = 1 \tag{9}$$

$$F = 0, \quad H = -Ro^{-1} = -\omega,$$
$$G = \rho_{-L}, \quad T = T_{-L} \quad \text{at}\ \xi = 0 \tag{10}$$

where the Rossby number is $Ro = \tilde{W}/\tilde{\omega}\tilde{L} = \omega^{-1}$ [5]. The dimensionless eqn (8) is the ideal-gas equation of state. Here the values of $\tilde{\rho}\tilde{\mu}$ and $\tilde{\rho}\tilde{\lambda}$ are taken constant due to $\tilde{\rho} \sim \tilde{T}^{-1}$, $\tilde{\lambda} \sim \tilde{T}$ and $\tilde{\mu} \sim \tilde{T}$ physically [6] and the specific heat $\tilde{c}_p$ is assumed to be a constant. The coefficient c in eqns (4)–(7) is given by

$$c = \int_{-1}^{1} \rho\,dz = 2 \Big/ \int_{0}^{1} T\,d\xi \tag{11}$$

Heat transfer from the high-temperature jet to the low-temperature one occurs at the interface plane and in the axial direction. The definition of interface plane here is a location

NOMENCLATURE

b coefficient defined in eqn (3)
c coefficient defined in eqn (11)
$\tilde{c}_p$ specific heat at constant pressure
F dimensionless function defined in eqn (3)
G dimensionless function defined in eqn (3)
h dimensionless function defined in eqn (3)
H dimensionless function defined in eqn (3)
$\tilde{L}$ half separation distance between jets
P dimensionless pressure, $P = \tilde{P}/\tilde{\rho}_L \tilde{W}^2$
$\tilde{P}$ pressure
Pr Prandtl number, $Pr = \tilde{c}_p \tilde{\mu}_L/\tilde{\lambda}_L$
q dimensionless heat flux (efficiency of heat transfer), $q = \tilde{q}/\tilde{\rho}_L \tilde{c}_p \tilde{W}\tilde{T}_L$
$\tilde{q}$ heat flux
$\tilde{q}_d$ heat flux in the limit of heat conduction
r dimensionless radial coordinate, $r = \tilde{r}/\tilde{L}$
Re_L Reynolds number, $Re_L = \tilde{\rho}_L \tilde{W}\tilde{L}/\tilde{\mu}_L$
Ro Rossby number, $Ro = \omega^{-1}$
T dimensionless temperature, $T = \tilde{T}/\tilde{T}_L$
$\tilde{T}$ temperature
u dimensionless radial velocity, $u = \tilde{u}/\tilde{W}$
$\tilde{u}$ radial velocity
v dimensionless circumferential velocity, $v = \tilde{v}/\tilde{W}$
$\tilde{v}$ circumferential velocity
w dimensionless axial velocity, $w = \tilde{w}/\tilde{W}$
$\tilde{w}$ axial velocity
$\tilde{W}$ axial velocity at jet exit
$\tilde{r}$ radial coordinate
r dimensionless radial coordinate, $r = \tilde{r}/\tilde{L}$
$\tilde{z}$ axial coordinate
z dimensionless axial coordinate, $z = \tilde{z}/\tilde{L}$.

Greek symbols
λ dimensionless thermal conductivity, $\lambda = \tilde{\lambda}/\tilde{\lambda}_L$
$\tilde{\lambda}$ thermal conductivity
μ dimensionless viscosity, $\mu = \tilde{\mu}/\tilde{\mu}_L$
$\tilde{\mu}$ viscosity
ξ transformed axial coordinate defined in eqn (2)
ρ dimensionless density, $\rho = \tilde{\rho}/\tilde{\rho}_L$
$\tilde{\rho}$ density
ω dimensionless jet angular speed, $\omega = \tilde{\omega}\tilde{L}/\tilde{W}$
$\tilde{\omega}$ jet angular speed
$\tilde{\omega}_r$ characteristic jet angular speed, $\tilde{\omega}_r = \tilde{W}_{Re_L=600}/\tilde{L}$
Ω dimensionless jet angular speed, $\Omega = \tilde{\omega}/\tilde{\omega}_r$.

Subscripts
d limit of heat conduction
L refers to upper jet
$-$L refers to lower jet
r reference angular speed
s interface plane.

Superscripts
$'$ differentiation with respect to ξ
$\tilde{}$ dimensional quantity
$\rightarrow$ velocity vector.

where the axial velocity approaches zero. Because the axial velocities of both opposed jets vanish at the interface plane, the heat transfer between two jets is readily evaluated by heat conduction. As a result, the heat flux between two jets equals

$$\tilde{q} = |\tilde{\lambda}(\partial \tilde{T}/\partial \tilde{z})_{\tilde{z}=\tilde{z}_s}| \quad (12)$$

The magnitude of $\tilde{q}$ can be also calculated according to the law of energy conservation.

$$2\pi \int_{\tilde{z}_s}^{\tilde{L}} \tilde{\rho}\tilde{c}_p \tilde{r}\tilde{u}\tilde{T}\,d\tilde{z} + \pi \tilde{r}^2 \tilde{q} = \tilde{\rho}_L \tilde{c}_p \pi \tilde{r}^2 \tilde{W}\tilde{T}_L \quad (13)$$

For convenience of physical presentations, we define a dimensionless heat flux $q = \tilde{q}/\tilde{\rho}_L \tilde{c}_p \tilde{W}\tilde{T}_L$ that means a ratio of the heat flux across the interface plane to the flux of thermal energy originally emerging from the hot upper jet. According to this dimensionless form, the magnitude of q can be physically viewed as the efficiency of heat transfer for the problem of interest. In terms of dimensionless variables (1), eqn (12) is expressed as

$$q = \frac{1}{c\,Pr\,Re_L} |(\partial T/\partial \xi)_{\xi=\xi_s}| \quad (14)$$

The problem governed by eqns (4)–(8) with boundary conditions (9) and (10) is a standard two-point boundary-value problem. It is solved numerically by means of an existing computer-library subroutine (BVPFD from *IMSL User's Manual*, 1989). The computational tolerance in all calculations is 10^{-6}. The numerical results are expressed in terms of the dimensionless spatial coordinate z according to the following relationship

$$z = c\int_0^{\xi} T\,d\xi - 1 \quad (15)$$

3. DISCUSSION AND CONCLUSIONS

From a physical point of view, the thermal energy is transported by two processes, namely heat conduction and heat convection. For convenience of physical interpretations, a case in the limit of heat conduction is considered first. In this limit, all velocities approach zero. Consequently the energy eqn (7) is reduced to

$$T'' = 0 \quad (16)$$

subject to the boundary conditions

$$T = 1 \quad \text{at } \xi = 1 \quad \text{and} \quad T = T_{-L} \quad \text{at } \xi = 0 \quad (17)$$

In this limit, the heat flux between two ends ($\tilde{q}_d$) is readily solved for a given problem. To make a comparison with the heat transfer for swirling jets, a ratio of heat flux $\tilde{q}/\tilde{q}_d$ as a function of ω (Ro^{-1}) is presented in Fig. 1. According to this figure, the magnitude of $\tilde{q}/\tilde{q}_d$ is invariably greater than unity and decreases with ω. The process of heat conduction plays a more important role in the transport of thermal energy for greater ω because the convective velocity in the axial direction near the interface plane ($z \approx 0$) is reduced as ω increases

[5]. With the same boundary conditions of temperature, the rate of heat transfer by heat conduction is less than that by heat convection in fluids such that the magnitude of $\tilde{q}/\tilde{q}_d$ is invariably greater than unity and decreases with ω. The dependence of q on ω is also illustrated in Fig. 1. The value of q decreases with ω, as expected physically. The efficiency of heat transfer from the high-temperature to the low-temperature swirling jet decreases as ω increases or the Rossby number decreases.

The efficiency of heat transfer q vs ω for $Pr = 0.7$, 1.0 and 1.3 is presented in Fig. 2. According to this figure, for fixed ω the efficiency of heat transfer between two swirling jets decreases with Pr. Physically, the rate of heat transfer decreases as the thermal diffusivity decreases. As a result, a smaller efficiency of heat transfer is expected for greater Pr.

From a practical point of view, the variation of heat transfer with the axial injection velocity ($\tilde{W}$) is of interest. According to the definitions, the magnitudes of both Re_L and Ro increase with the axial injection velocity for a given fluid. The effect of Re_L on q for $Re_L = 300$ and 600 is illustrated in Fig. 3. Because the axial injection velocity is involved in the definition of Ro, a dimensionless rotating speed is defined as $\Omega = \tilde{\omega}/\tilde{\omega}_r$ in Fig. 3, where the characteristic jet angular speed $\tilde{\omega}_r$ ($= \tilde{W}_{Re_L=600}/\tilde{L}$) is a reference speed. The results reveal that the magnitude of q for $Re_L = 300$ is greater than that for $Re_L = 600$ for smaller Ω whereas a converse behavior occurs for $\Omega > 0.65$ according to Fig. 3. The physical reasons for this converse phenomenon are as follows. The axial injection velocity $\tilde{W}$ for $Re_L = 300$ is half that for $Re_L = 600$ for the given $\tilde{L}$ and fluid. Thus, with increasing the jet angular velocity the reduction ratio of axial velocity is expected to be greater for smaller $\tilde{W}(Re_L)$. Because the effect of heat convection on heat transfer decreases with this reduction ratio, there is an intersection of q for $Re_L = 300$ and 600 in Fig. 3. From a viewpoint of heat transfer, the efficiency of heat transfer for smaller Re_L is better than that for greater Re_L only when the jet angular speed is small.

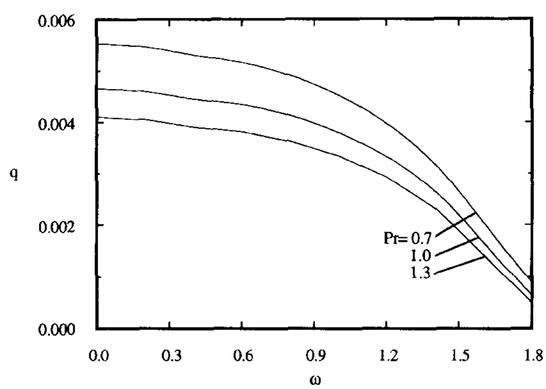

Fig. 2. Efficiency of heat transfer q vs ω for $Pr = 0.7$, 1.0 and 1.3 ($Re_L = 600$ and $T_{-L} = 0.8$).

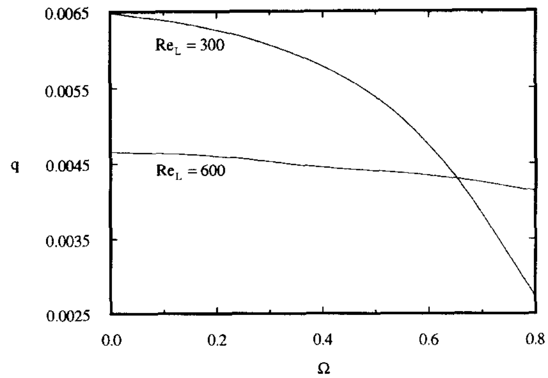

Fig. 3. Efficiency of heat transfer q vs Ω for $Re_L = 300$ and 600 ($Pr = 1$ and $T_{-L} = 0.8$).

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